APPENDIX. NON-SI SYSTEMS OF QUANTITIES AND UNITS

Although the Système International is the recommended system for representing quantities and units, a great deal of the existing literature in physics has been expressed in terms of older systems. It is thus necessary to understand the relationship between SI and these systems if the older literature is to be fully utilized. The discussion here is not intended to be a complete review of these systems, nor to advance their use; its only purpose is to provide a basis for their translation into SI.

A.1 Systems of equations with three base quantities

During the 19th century, when physics was dominated by Newtonian mechanics, electromagnetism was forced into an artificially restrictive three-dimensional framework. As a consequence, at least three different systems have been developed from the base quantities length, mass and time:

1a. The "electrostatic" system defines electric charge to be a derived quantity based on Coulomb's law for the force between two electric charges,

$$\boldsymbol{F} = k_{\mathrm{e}} \frac{q_1 q_2 \boldsymbol{r}}{\epsilon r^3}, \tag{1}$$

by choosing $k_{\mathrm{e}} = 1$ and defining the permittivity ϵ to be a dimensionless quantity, taking its value to be unity for a vacuum.

1b. The "electromagnetic" system defines electric current to be a derived quantity based on Ampère's law for the force between two electric current elements,

$$\mathrm{d}^2\boldsymbol{F} = k_{\mathrm{m}} \mu \frac{i_1 \mathrm{d}\boldsymbol{l}_1 \times (i_2 \mathrm{d}\boldsymbol{l}_2 \times \boldsymbol{r})}{r^3}, \tag{2}$$

by choosing $k_{\mathrm{m}} = 1$ and defining the permeability μ to be a dimensionless quantity, taking its value to be unity for a vacuum.

1c. The "symmetrical" Gaussian system uses electric quantities (including electric current) from system (1a) and magnetic quantities from system (1b).

In systems (1a) and (1b) a factor of the square of the speed of light in vacuum appears explicitly in some of the equations among quantities. In system (1c) the first power of the speed of light appears in many of the equations relating electric and magnetic quantities.

These systems are "non-rationalized" because the choices $k_{\mathrm{e}} = 1$ and $k_{\mathrm{m}} = 1$ in eqs. (1) and (2) leads to the appearance of factors of 2π and 4π in situations that involve plane geometry, and to their absence in situations that

Table 12. Non-rationalized and rationalized systems.

Non-rationalized symmetrical (Gaussian) system with three base quantities (1.c)	Rationalized system with four base quantities
Equations	
$c\boldsymbol{\nabla} \times \boldsymbol{E}^* = -\partial \boldsymbol{B}^*/\partial t$	$\boldsymbol{\nabla} \times \boldsymbol{E} = -\partial \boldsymbol{B}/\partial t$
$c\boldsymbol{\nabla} \times \boldsymbol{H}^* = 4\pi \boldsymbol{j}^* + \partial \boldsymbol{D}^*/\partial t$	$\boldsymbol{\nabla} \times \boldsymbol{H} = \boldsymbol{j} + \partial \boldsymbol{D}/\partial t$
$\boldsymbol{\nabla} \cdot \boldsymbol{D}^* = 4\pi \rho^*$	$\boldsymbol{\nabla} \cdot \boldsymbol{D} = \rho$
$\boldsymbol{\nabla} \cdot \boldsymbol{B}^* = 0$	$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$
$\boldsymbol{F} = q^*(\boldsymbol{E}^* + \boldsymbol{v} \times \boldsymbol{B}^*/c)$	$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$
$w = (\boldsymbol{E}^* \cdot \boldsymbol{D}^* + \boldsymbol{B}^* \cdot \boldsymbol{E}^*)/8\pi$	$w = \frac{1}{2}(\boldsymbol{E} \cdot \boldsymbol{D} + \boldsymbol{B} \cdot \boldsymbol{E})$
$\boldsymbol{S} = c(\boldsymbol{E}^* \times \boldsymbol{H}^*)/4\pi$	$\boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{H}$
$\boldsymbol{E}^* = -(\boldsymbol{\nabla} V^* + (1/c)\partial \boldsymbol{A}^*/\partial t)$	$\boldsymbol{E} = -(\boldsymbol{\nabla} V + \partial \boldsymbol{A}/\partial t)$
$\boldsymbol{B}^* = \boldsymbol{\nabla} \times \boldsymbol{A}^*$	$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$
$\boldsymbol{D}^* = \epsilon_{\mathrm{r}} \boldsymbol{E}^*$	$\boldsymbol{D} = \epsilon \boldsymbol{E} = \epsilon_{\mathrm{o}} \epsilon_{\mathrm{r}} \boldsymbol{E}$
$= \boldsymbol{E}^* + 4\pi \boldsymbol{P}^*$	$= \epsilon_{\mathrm{o}} \boldsymbol{E} + \boldsymbol{P}$
$\boldsymbol{B}^* = \mu_{\mathrm{r}} \boldsymbol{H}^*$	$\boldsymbol{B} = \mu \boldsymbol{H} = \mu_{\mathrm{o}} \mu_{\mathrm{r}} \boldsymbol{H}$
$= \boldsymbol{H}^* + 4\pi \boldsymbol{M}^*$	$= \mu_{\mathrm{o}}(\boldsymbol{H} + \boldsymbol{M})$
$\epsilon_{\mathrm{r}} = 1 + 4\pi \chi_{\mathrm{e}}^*$	$\epsilon_{\mathrm{r}} = 1 + \chi_{\mathrm{e}}$
$\mu_{\mathrm{r}} = 1 + 4\pi \chi_{\mathrm{m}}^*$	$\mu_{\mathrm{r}} = 1 + \chi_{\mathrm{m}}$
Physical constants	
$\alpha = e^{*2}/\hbar c$	$\alpha = e^2/4\pi\epsilon_{\mathrm{o}}\hbar c = \mu_{\mathrm{o}} c e^2/2h$
$a_{\mathrm{o}} = \hbar^2/m_{\mathrm{e}} e^{*2}$	$a_{\mathrm{o}} = 4\pi\epsilon_{\mathrm{o}}\hbar^2/m_{\mathrm{e}} e^2$
$hcR_\infty = e^{*2}/2a_{\mathrm{o}}$	$hcR_\infty = e^2/8\pi\epsilon_{\mathrm{o}} a_{\mathrm{o}}$
$r_{\mathrm{e}} = e^{*2}/m_{\mathrm{e}} c^2$	$r_{\mathrm{e}} = \mu_{\mathrm{o}} e^2/m_{\mathrm{e}}$
$\mu_{\mathrm{B}}^* = e^* \hbar/2m_{\mathrm{e}} c$	$\mu_{\mathrm{B}} = e\hbar/2m_{\mathrm{e}}$
$\omega_{\mathrm{L}} = (q^*/2mc)B^*$	$\omega_{\mathrm{L}} = (q/2m)B$
$\gamma^* = gI(e^*/mc)$	$\gamma = gI(e/m)$

have cylindrical or spherical symmetry where these factors might normally be expected. On the other hand, if the factors k_{e} and k_{m} are set equal to $1/4\pi$ in eqs. (1) and (2), respectively (recognizing the spherical symmetry of these equations), then the factors of 2π and 4π appear explicitly only in those equations where they would be expected from the geometry of the system. In this form the equations are said to be "rationalized".

A.2 Systems of equations with four base quantities

The system of quantities is enlarged to four dimensions by including an electrical quantity as a fourth base quantity. In SI and in its older relative, the MKSA system, the fourth quantity is taken to be electric current, and in the Système International eqs. (1) and (2) are rationalized ($k_e = k_m = 1/4\pi$). As a result, permeability μ and permittivity ϵ are dimensional physical quantities. If electrostatics and electrodynamics are to be coherent, thus avoiding the explicit introduction of the factor c asymmetrically into the expressions for electric and magnetic quantities, ϵ_o and μ_o must satisfy the condition

$$\epsilon_o \mu_o c^2 = 1.$$

In SI the permeability of vacuum μ_o is defined to have the value

$$\mu_o = 4\pi \times 10^{-7}\,\mathrm{N/A^2} = 4\pi \times 10^{-7}\,\mathrm{H/m}.$$

A.3 Relations between quantities in different systems

The basic equations between quantities in the non-rationalized symmetrical (Gaussian) system (1c) and the corresponding equations in the rationalized four-dimensional system are given in table 12. In order to distinguish the physical quantities in the two systems, those in the three-dimensional system are indicated with an asterisk (*) when they differ from their corresponding quantities of the rationalized four-dimensional system.+ The relationships between the two sets of quantities are determined by setting $X^* = a_X X$ in the first column and comparing the resultant equations with the corresponding ones in the second column. These substitutions lead to:

$$(4\pi\epsilon_o)^{\frac{1}{2}} = \frac{E^*}{E} = \frac{V^*}{V} = \frac{Q}{Q^*} = \frac{\rho}{\rho^*} = \frac{j}{j^*} = \frac{I}{I^*} = \frac{P}{P^*},$$

$$(4\pi/\epsilon_o)^{\frac{1}{2}} = \frac{D^*}{D},$$

$$(4\pi\mu_o)^{\frac{1}{2}} = \frac{H^*}{H},$$

$$(4\pi/\mu_o)^{\frac{1}{2}} = \frac{B^*}{B} = \frac{A^*}{A} = \frac{M}{M^*},$$

$$4\pi = \frac{\chi_e}{\chi_e^*} = \frac{\chi_m}{\chi_m^*}.$$

A.4 The CGS system of units

The centimetre-gram-second (CGS) system of units is a coherent system based on the three base units: centimetre, gram and second. These base units

+ Symbols for Gaussian quantities may also be distinguished from those for the four-dimensional quantities by the superscript ^{s} or subscript $_s$ (for symmetric) instead of the asterisk.

Table 13. CGS base units and derived units with special names.

Quantity *Grandeur*	Unit; *Unité* Name *Nom*	Symbol *Symbole*	Expression in terms of base units *Expression en unités de base*
length *longueur*	centimetre *centimètre*	cm	
mass *masse*	gram *gramme*	g	
time *temps*	second *seconde*	s	
force; *force*	dyne	dyn	$\mathrm{cm\,g\,s^{-2}}$
energy; *énergie*	erg	erg	$\mathrm{cm^2\,g\,s^{-2}}$
viscosity; *viscosité*	poise	P	$\mathrm{cm^{-1}\,g\,s^{-1}}$
kinematic viscosity; *viscosité cinématique*	stokes	St	$\mathrm{cm^2\,s^{-1}}$
acceleration of free fall; *accélération de la pesanteur* [a]	gal	Gal	$\mathrm{cm\,s^{-2}}$

[a] The gal is a unit used in geophysics to express the earth's gravitational field; it should not be used as a unit of acceleration other than in this specific sense.

and their symbols, as well as the names and symbols of derived units having special names in the CGS system are given in table 13.

The CGS "electrostatic" system of units (esu) forms a coherent system of units in combination with the three-dimensional "electrostatic" system of quantities of (1a). In its less common form as a four-dimensional system, the electrostatic unit of charge (sometimes called the franklin; symbol, Fr) is introduced and the permittivity of vacuum is set equal to $\epsilon_o = 1\,\mathrm{Fr^2\,dyn^{-1}\,cm^{-2}}$. Other units may then be derived using the usual rules for constructing a coherent set of units from a set of base units.

The CGS "electromagnetic" system of units (emu) forms a coherent system of units in combination with the three-dimensional "electromagnetic" system of quantities of (1b). In its four-dimensional form, the fourth base unit is taken to be the current unit, abampere (symbol, abamp), by defining the permeability

Table 14. CGS magnetic units with special names.

Quantity *Grandeur*	Unit; *Unité* Name *Nom*	Symbol *Symbole*	Dimension[a]	Equivalence between CGS units and corresponding 4-dimensional SI units
H^*	oersted	Oe	$L^{-\frac{1}{2}} M^{\frac{1}{2}} T^{-1}$	$\frac{1}{4\pi}$ abamp/cm $= 10^{-4}$ T/$\mu_\circ$
B^*	gauss	G, (Gs)	$L^{-\frac{1}{2}} M^{\frac{1}{2}} T^{-1}$	10^{-4} T
Φ^*	maxwell	Mx	$L^{\frac{3}{2}} M^{\frac{1}{2}} T^{-1}$	10^{-8} Wb
F_m^*	gilbert	Gi, (Gb)	$L^{\frac{1}{2}} M^{\frac{1}{2}} T^{-1}$	$\frac{1}{4\pi}$ abamp $= 10^{-6}$ T m/$\mu_\circ$

[a] L = length; M = mass; T = time.

of vacuum to be $\mu_\circ = 1$ g cm s^{-2} abamp^{-2}. The force between two parallel infinitely long wires, 1 cm apart in vacuum, each carrying a current of 1 abamp, is 2 dyn per cm of length.

The "mixed", "symmetrized", or "Gaussian" CGS units, consisting of the set of electric units of the esu system and the magnetic units of the emu system, form a coherent system of units when used in combination with the three-dimensional "symmetrical system" or "Gaussian system" of equations (1c).

Special names and symbols have been given to four of the magnetic emu or Gaussian CGS units. These are given in table 14. In evaluating the relationship between a CGS non-rationalized unit and an SI rationalized unit one must include not only the transformation of the ***quantities*** given in the preceding section but also the transformation of the ***units*** from centimetre and gram to metre and kilogram. In addition, the relationship between a four-dimensional unit involving the ampere and its corresponding three-dimensional unit includes the quantity $\mu_\circ$, recognizing that its value is unity in the latter system.

The CGS system enlarged by the kelvin (K) as unit of thermodynamic temperature, and by the mole (mol) as unit of amount of substance or by the candela (cd) as unit of luminous intensity has been used in thermodynamics and photometry, respectively. The two units in the field of photometry derived from cm, g, s, cd and sr that have been given special names and symbols are listed in table 15.

A.5 Atomic units

It is often appropriate in theoretical physics and in numerical computations to use a system of "dimensionless" quantities obtained by setting the numerical values of $\hbar$, c and either m_e or m_u equal to unity. It is more correct, however, to maintain the description of section **1** and to treat this as a unit system in which the units are fundamental physical quantities rather than arbitrary

Table 15. CGS units in photometry with special names.

Quantity *Grandeur*	Derived unit; *Unité derivée* Name *Nom*	Symbol *Symbole*	Expression *Expression*
luminance; *luminance*	stilb	sb	cm^{-2} cd
illuminance; *éclairement lumineux*	phot	ph	cm^{-2} cd sr

artifacts such as the metre or the second. It is, in fact, strongly recommended that physical computations be carried out and reported in terms of such units in order that the results should be independent (to the greatest possible extent) of any uncertainties in the values of the physical constants.

The standard choice of units in quantum electrodynamics takes $\hbar$ and c as the units of action and velocity respectively, so that the elementary charge is $(4\pi\epsilon_\circ\alpha)^{\frac{1}{2}}$ (charge units) where the fine-structure constant α is the natural measure of the electromagnetic interaction.

For computations in atomic and molecular physics a more appropriate choice (known as 'atomic units' or 'au') takes the electron mass m_e to be the unit of mass, the Bohr radius, $a_\circ = \hbar/(m_\mathrm{e}c\alpha)$ to be the unit of length and $\hbar/(m_\mathrm{e}c^2\alpha^2)$ to be the unit of time. As a result the unit of velocity is αc and the unit of energy is $E_\mathrm{h} = m_\mathrm{e}c^2\alpha^2 = 2R_\infty hc$, which has been given the name 'hartree'. The atomic units form an unrationalized, three-dimensional coherent system with $\epsilon_\circ$ set equal to unity and the elementary charge e as the unit of charge.

Since atomic units are natural physical quantities rather than artificial constructs, it is appropriate to write them in italic (***sloping***) type rather than in the roman (upright) type normally used for units: the physical quantities are represented as multiples of physical constants.